PHYS 101 – General Physics I Midterm Exam

Duration: 150 minutes Saturday, 12 April 2023; 12:00

1. A car starts from rest and moves with constant acceleration $a = 2\ \mathrm{m/s^2}$ until it reaches the speed $26\ \mathrm{m/s}$. It then continues to move with this constant speed.

(a) (6 Pts.) For how long does the car accelerate?

(b) (7 Pts.) For how much longer should the car travel with constant speed until its average speed becomes $24\ \mathrm{m/s}$?

(c) (7 Pts.) What will be the total distance covered by the car from start until the average speed becomes $24\ \mathrm{m/s}$?

Solution:

(a)

$$v(t) = v_0 + a\,t_1 \;\rightarrow\; 26\ \mathrm{m/s} = (2\ \mathrm{m/s^2})t_1 \;\rightarrow\; t_1 = 13\ \mathrm{s}\,.$$

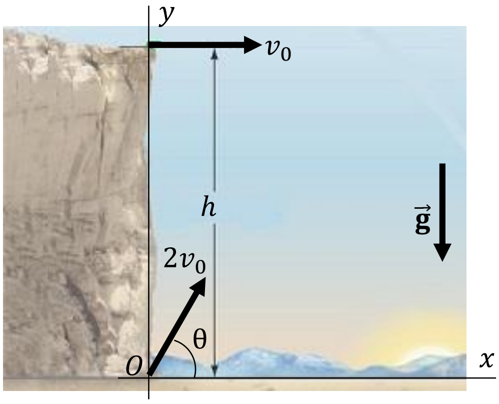
(b) During this time the car travels $s_1 = \frac{1}{2}at_1^2 \;\rightarrow\; \frac{1}{2}(2\ \mathrm{m/s^2})(13\ \mathrm{s})^2 \;\rightarrow\; s_1 = 169\ \mathrm{m}$. If the car travels with constant speed $26.0\ \mathrm{m/s}$ for time t_2, the distance it travels during this time will be $s_2 = (26.0\ \mathrm{m/s})t_2$. Average speed $\bar{s}$ is defined as

$$\bar{s} = \frac{s_1 + s_2}{t_1 + t_2} \;\rightarrow\; (24\ \mathrm{m/s}) = \frac{169\,m + (26.0\ \mathrm{m/s})t_2}{(13\ \mathrm{s}) + t_2} \rightarrow t_2 = 71.5\ \mathrm{s}\,.$$

(c) Total distance covered during this time is

$$s_1 + s_2 = (169\ \mathrm{m}) + (26.0\ \mathrm{m/s})(71.5\ \mathrm{s}) = 2028\ \mathrm{m}$$

2. A projectile is shot at time $t = 0$ horizontally with initial speed v_0 from the top of a cliff which is a distance h above ground level. At the same time, another projectile is shot from the base of the cliff with initial speed $2v_0$, at an angle $\theta = \pi/3$ with the horizontal. It is observed that the two projectiles collide in midair. Answer the following questions using the coordinate system shown in the figure, and expressing your results in terms of g, h and v_0. $(\cos(\pi/3) = \cos 60° = 1/2)$



(a) (5 Pts.) At what time does the collision occur?

(b) (5 Pts.) What is the position vector of the point where the collision occurs?

(c) (5 Pts.) What is the relative speed of the two projectiles with respect to each other just before the collision?

(d) (5 Pts.) Given h, for what range of values of v_0 the projectiles do not collide?

Solution:

(a) Kinematical equations describing positions of the two projectiles are

$$\vec{\mathbf{r}}_1 = (v_0 t)\,\hat{\mathbf{i}} + \left(h - \frac{1}{2}\mathrm{g}\,t^2\right)\hat{\mathbf{j}}\,, \qquad \vec{\mathbf{r}}_2 = (2\,v_0 \cos\theta\ t)\,\hat{\mathbf{i}} + \left(2v_0 \sin\theta\, t - \frac{1}{2}g\,t^2\right)\hat{\mathbf{j}}.$$

$$\cos\theta = \frac{1}{2} \rightarrow \sin\theta = \frac{\sqrt{3}}{2} \rightarrow \vec{\mathbf{r}}_2 = (v_0\,t)\,\hat{\mathbf{i}} + \left(\sqrt{3}v_0\,t - \frac{1}{2}g\,t^2\right)\hat{\mathbf{j}}\,.$$

Collision occurs when $\vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_2 \rightarrow \quad t_c = h/(\sqrt{3}\,v_0)\,.$

(b) Collision occurs at the point $\vec{\mathbf{r}}_c = \vec{\mathbf{r}}_1(t_c) = \vec{\mathbf{r}}_2(t_c)$.

$$\vec{\mathbf{r}}_c = \left(\frac{h}{\sqrt{3}}\right)\hat{\mathbf{i}} + h\left(1 - \frac{\mathrm{g}h}{6v_0^2}\right)\hat{\mathbf{j}}\,.$$

(c)

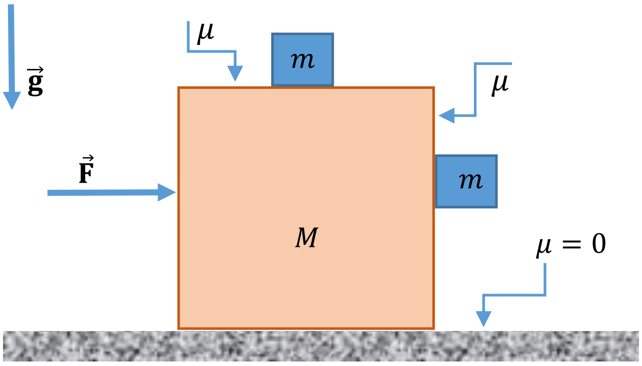
$$\frac{d\vec{\mathbf{r}}_1}{dt} = \vec{\mathbf{v}}_1 = (v_0)\,\hat{\mathbf{i}} - (\mathrm{g}\,t)\,\hat{\mathbf{j}}\,, \qquad \frac{d\vec{\mathbf{r}}_2}{dt} = \vec{\mathbf{v}}_2 = (v_0)\,\hat{\mathbf{i}} + \left(\sqrt{3}v_0 - \mathrm{g}\,t\right)\hat{\mathbf{j}}\,.$$

$$\vec{\mathbf{v}}_{21} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 \rightarrow \vec{\mathbf{v}}_{21} = \left(\sqrt{3}v_0\right)\hat{\mathbf{j}} \rightarrow v_{21} = \sqrt{3}v_0\,.$$

(d) No collision can occur if the y component of the collision point is negative (i.e., below the ground level).

$$\left(1 - \frac{\mathrm{g}h}{6v_0^2}\right) < 0 \rightarrow v_0 < \sqrt{gh/6}\,.$$

3. A solid cube of mass M is on a horizontal frictionless table. Two smaller blocks of equal mass m are placed on the top and the side of the cube as shown in the figure. Coefficients of static and kinetic between the small blocks and the cube are both μ. A horizontal force of magnitude F is applied to the cube as shown in the figure. Magnitude of the gravitational acceleration is g. Assume $\mu < 1$.

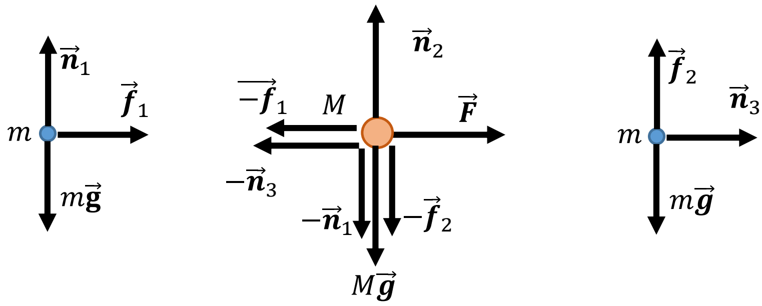


(a) (6 Pts.) Draw a free body diagram for all three objects.

(b) (7 Pts.) What is the minimum value of F so that the small block on the side does not slide down?

(c) (7 Pts.) What is the maximum value of F so that the small block on the top does not slide with respect to the cube?

Solution: (a)



(b) Writing Newton's second law for the top block, we find the not slipping condition as

$$f_1 = ma, n_1 = mg, f_1 \leq \mu mg \;\rightarrow\; a \leq \mu\, g\,.$$

Writing Newton's second law for the side block, we find the not slipping condition as

$$n_3 = ma\,, \quad f_2 = mg\,, \quad f_2 \leq \mu ma \;\rightarrow\; a \geq \frac{g}{\mu}.$$

For $\mu < 1$ both conditions can not be satisfied simultaneously. If the side block does not slide, its acceleration is the same as that of the cube, say a_s, the top block has no acceleration in the vertical direction and, due to the kinetic friction force, its horizontal acceleration is a_t. Newton's second law applied to the top block gives

$$f_1 = ma_t\,, \quad n_1 - mg = 0\,, \quad f_1 = \mu n_1 = \mu mg \;\rightarrow\; a_t = \mu g\,.$$

Newton's second law for the cube implies

$$F - f_1 - n_3 = Ma \;\rightarrow\; F = (m+M)a + \mu mg\,, \quad a \geq \frac{g}{\mu} \;\rightarrow\; F_{\text{min}} = (m+M)\frac{g}{\mu} + \mu mg\,.$$

(c) When $F = F_{\text{max}}$ and the top block does not to slide, its acceleration is the same as that of the cube. The side block is sliding down the side of the cube, but its acceleration in the horizontal direction is the same as that of the cube and the top block. Newton's second law implies

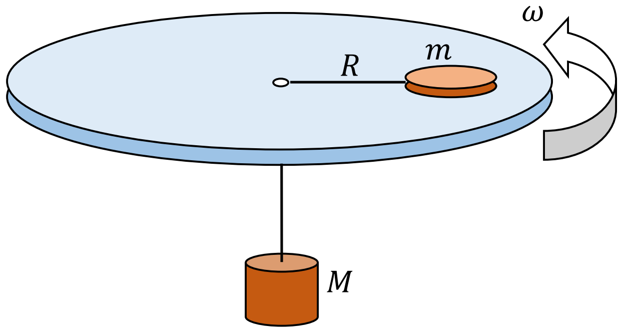
$$n_3 = ma\,, \quad f_1 = ma\,, \quad F - f_1 - n_3 = Ma \rightarrow F = (2m+M)a\,, \quad a \leq \mu\, g \rightarrow F_{\text{max}} = (2m+M)\mu g.$$

4. A horizontal disk is rotating around its center with constant angular velocity $\omega = 4.0$ rad/sec. A puck of mass $m = 1.0$ kg is sitting on the disk, and is rotating with it. The mass is situated at a distance $R = 0.5$ m away from the center of the disk. This mass is connected to another mass M dangling in air by a massles string that goes through a small hole at the center of the disk without friction. Gravitational acceleration is $\mathrm{g} = 10.0\ \mathrm{m/s^2}$, and the coefficient of static friction between the disk and the mass is $\mu_s = 0.4$.

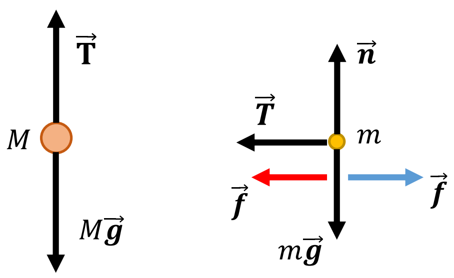
(a) (4 Pts.) Draw a free body diagram for both masses.

(b) (8 Pts.) What is the minimum value of M so that m does not start sliding outward?

(c) (8 Pts.) What is the maximum value of M so that m does not start sliding inward?



Solution: (a)



Direction of the static friction force depends on whether m is tending to slide inward or outward.

Newton's second law implies

$$T - M\mathrm{g} = 0\,, \qquad T \pm f = mR\omega^2\,, \qquad n - m\mathrm{g} = 0\,, \qquad f \le \mu n \;\rightarrow\; f \le \mu m\mathrm{g}.$$

(b) For the minimum value of M, we have

$$M\mathrm{g} + f = mR\omega^2 \;\rightarrow\; f = mR\omega^2 - M\mathrm{g} \;\rightarrow\; mR\omega^2 - M\mathrm{g} \le \mu m\mathrm{g} \;\rightarrow\; M \ge m\left(\frac{R\omega^2}{\mathrm{g}} - \mu\right).$$

This means $M_{\min} = 0.4$ kg.

(c) For the maximum value of M, we have

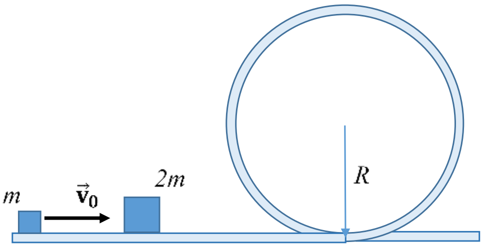
$$M\mathrm{g} - f = mR\omega^2 \;\rightarrow\; f = M\mathrm{g} - mR\omega^2 \;\rightarrow\; M\mathrm{g} - mR\omega^2 \le \mu m\mathrm{g} \;\rightarrow\; M \le m\left(\frac{R\omega^2}{\mathrm{g}} + \mu\right).$$

This means $M_{\max} = 1.2$ kg.

5. A block of mass m is moving with speed v_0 towards a stationary block of mass $2m$ on a frictionless track. There is a vertical circular loop of radius R on the frictionless track after the $2m$ mass. The gravitational acceleration is g.

(a) (10 Pts.) Find the velocities of both masses after they undergo an elastic collision.

(b) (10 Pts.) What is the minimum velocity v_0 so that the $2m$ block can pass through the vertical loop without losing contact with the track?



Solution:

(a) Momentum is conserved during the collision.

$$p_i = p_f \;\rightarrow\; mv_0 = mv_1' + 2mv_2' \,.$$

Relative speed of the colliding blocks does not change during an elastic collision, meaning $v_0 = -(v_1' - v_2')$. Solving these two equations, we get

$$v_1' = -\frac{1}{3}v_0 \,, \qquad v_2' = \frac{2}{3}v_0 \,.$$

(b) To pass through the vertical loop without losing contact with the track, the $2m$ block should have a minimum speed $v_{T\min} = \sqrt{gR}$ at the top of the track. Since mecanical energy is conserved, we have

$$\frac{1}{2}(2m)\left(\frac{2}{3}v_{0\min}\right)^2 = \frac{1}{2}(2m)(gR) + (2m)g(2R) \;\rightarrow\; v_{0\min} = \frac{3}{2}\sqrt{5gR} \,.$$